Multiport conversions between S, Z, Y, h, ABCD, and T parameters

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Introduction
This poster presents generalized formulation for conversions between electrical parameters matrices. The calculations for two-port devices, presented in [1], are here extended for multiport systems.

Definitions
Submatrices partitioning: The multiport approach consists on considering the S-parameters matrix of the device of interest as a partition of 4 submatrices. Ports are divided in 2 groups: external and internal ports. This kind of partitioning, illustrated here on a 5 matrix can be applied to Z, Y, h, ABCD and T matrices as well.

Electrical parameters: Matrices defined in [1] can be expanded to multiport as follow:

\[
\begin{align*}
\begin{bmatrix}
V_1 \\
V_2 \\
I_1 \\
I_2 \\
Y_{11} & Y_{12} & Y_{13} \\
Y_{21} & Y_{22} & Y_{23} \\
I_{11} & I_{12} & I_{13} \\
I_{21} & I_{22} & I_{23}
\end{bmatrix}
&= \begin{bmatrix}
Z_{ee} & Z_{ei} & Z_{en} \\
Z_{ie} & Z_{ii} & Z_{in} \\
Z_{le} & Z_{li} & Z_{ll}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
I_1 \\
I_2
\end{bmatrix}
\end{align*}
\]

Transfert parameters matrix [T]:

\[
\begin{align*}
\begin{bmatrix}
T_{11} & T_{12} & T_{13} \\
T_{21} & T_{22} & T_{23} \\
T_{31} & T_{32} & T_{33}
\end{bmatrix}
&= \begin{bmatrix}
Z_{ee} & Z_{ei} & Z_{en} \\
Z_{ie} & Z_{ii} & Z_{in} \\
Z_{le} & Z_{li} & Z_{ll}
\end{bmatrix}
\begin{bmatrix}
S_{1e} & S_{1i} & S_{1n} \\
S_{2e} & S_{2i} & S_{2n} \\
S_{3e} & S_{3i} & S_{3n}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
I_1 \\
I_2
\end{bmatrix}
\end{align*}
\]

Chain matrices [ABCD] and [T] are properly defined only when the system is balanced (same number of internal and external ports). If the system is unbalanced, we have to ensure the uniqueness of the solution and then apply the pseudo-inverse operator instead of the inverse matrix function.

S-parameters Normalization
The change of reference impedance of a multiport S-parameters matrix [Z] to [Z] is:

\[
[S'] = [A]^{-1}([S] - [p]I),([I] - [p])^{-1}[A]'
\]

where

\[
\begin{align*}
A &= [G_1^{-1}, [G_3], [I] - [p]]^{-1} \\
[p] &= [Z_e] - [Z_i],[Z_e] + [Z_i]^{-1}
\end{align*}
\]

[I] is the identity matrix, [G_1], [G_3], [Z_e] and [Z_i] are previously defined. All matrices, except [S] and [S'], are diagonal matrices.

Embedding
The embedding procedure, and SnP reduction, consist on the S-parameters calculation at the external ports according to the perfectly known S-parameters connected at the internal ports. The formula is:

\[
[S_{Global}] = [S_{ee}] + [S_{ei}]([I] - [p])^{-1}[S_{ie}]
\]

We can apply the same embedding procedure with [Z] or [Y] matrices without any condition on the multiport device:

\[
[Z_{Global}] = [Z_{ee}] + [Z_{ei}]([I] - [p])^{-1}[Z_{ie}]
\]

We can calculate the same embedding procedure with [T] matrix:

\[
[T_{Global}] = [T_{ee}] + [T_{ei}]([I] - [p])^{-1}[T_{ie}]
\]

De-embedding
The de-embedding procedure consists on extracting the [S] when [S] and [S_{Global}] are known:

\[
[S_{Global}] = [S_{ee}] + [S_{ei}]([I] - [p])^{-1}[S_{ie}]
\]

This equation works only when [S] describes a balanced system. If the number of internal ports (N_i) is lower than the number of external ports (N_e), we can use the pseudo-inverse operator to invert S_{Global} and get the least-square solution. Otherwise, the solution can not be solved and requires more assumptions.

This table illustrates a de-embedding on a multiport device. The first line represents a balanced system. The second line is an unbalanced system with N_i > N_e. The matrix inversion has to be done with the pseudo-inverse operator. The third line represents an unbalanced system with N_i < N_e. There is not uniqueness of the solution for the matrix inversion. Note that S to T matrix transformation is lossy [3] with unbalanced networks when N_i > N_e.

Matrices Conversions

\[
[Y] = [Z]^{-1} = [A][C]^{-1} + [A][C]^{-1}[B][D]^{-1}[A]
\]

S-Parameter and T-Parameter Conversion With Symmetry

\[
[S_{Global}] = [S_{ee}] + [S_{ei}]([I] - [p])^{-1}[S_{ie}]
\]

\[
[T_{Global}] = [T_{ee}] + [T_{ei}]([I] - [p])^{-1}[T_{ie}]
\]

Conclusion
This poster presents a multiport extension of the matrix parameters formulas by Frickey in 1994 [1]. The reader can handle, convert, normalize or reduce any matrix representing a linear multiport.

References

Downloads