Stability Issues in the Design of High Power Amplifiers and Oscillators

S. Mons\textsuperscript{1}, T. Reveyrand\textsuperscript{1}, T. Gasseling\textsuperscript{2}, JC. Nallatamby\textsuperscript{1}, R. Quéré\textsuperscript{1}, E. Ngoya\textsuperscript{1}

(1) Xlim Lab, University of Limoges - 123, Av. Thomas 87060 Limoges, France
(2) Amcad Engineering - Ester 87000 Limoges, France

📞: +33 555-457296 ☏: +33 555-457666
✉️: sebastien.mons@xlim.fr
http://www.xlim.fr
Outline

Small-Signal Stability Analysis
K-Factor, Normalized Determinant Function, Characteristic Determinant

Large Signal Analysis
NDF, Open loop analysis, Oscillator analysis

Measurement
Hot S parameters, High impedance probes

RF-SOC problematic
Full nodal stability analysis

Conclusion
A Stability criterion for designer
Motivation

Assessing Stability is a difficult task from the theoretical point of view !!
Checking stability before fabrication is mandatory for RF engineers !!
As performances demand increases, assuring robustly stable behavior will be a key point for commercial CAD tools…
…proposed solutions must be rigorous, low time consuming and compatibles with CAD utilities

Practical solutions

Efficient simulation methods based on perturbation theory have already been proposed for stability prediction
Characterization methods have been investigated for stability predicting
**Small Signal Analysis :** *K*-factor Criterion

**Origin**  
A linear quadripole is unconditionally stable if it does not reflect a higher power than the received one, for any passive termination.

**Definition**

\[
\begin{align*}
|\Gamma_L| < 1 & \Rightarrow |\Gamma_{\text{IN}}| < 1 \\
|\Gamma_S| < 1 & \Rightarrow |\Gamma_{\text{OUT}}| < 1 \\
K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{21}.S_{12}|} & > 1 \\
|\Delta| = |S_{21}.S_{12} - S_{11}.S_{22}| & > 1
\end{align*}
\]

**Use**

- \( K > 1 \) & \( |\Delta| < 1 \) → unconditional stability
- \( K < 1 \) → conditional stability

**Limitations**

Based on the reduction of a complex circuit → a pole-zero compensation is still possible

Criterion validity → Stability of stable unloaded circuit


Small Signal Analysis: Open Loop Analysis

Return Difference Function (One device circuit)

1 - Direct Resolution:
\[ RD = \frac{\Delta}{\Delta_o} \quad \text{with} \quad \Delta_o = \Delta \text{ (active sources off)} \]

2 - Undirect resolution:
\[ RD = 1 + RR \]

\[ RD \text{ cannot have PRP poles} \quad \text{Nyquist analysis} \]

Normalized Determinant Function (N-devices circuit)

1 - Direct Resolution:
\[ \text{NDF} = \frac{\Delta}{\Delta_{ON}} \quad \text{NDF cannot have PRP poles} \quad \text{Nyquist analysis} \]

2 - Undirect resolution:
\[ \text{NDF} = \prod_{i=1}^{n} (RD_i) \quad \text{With} \quad RD_i = \frac{\Delta}{\Delta_{oi}} \quad \Delta_{oi} = \Delta(gm_o \ldots gm_{i-1}=0) \]

Small Signal Analysis: Open loop analysis

Practical evaluation

Possible States of the transistors

<table>
<thead>
<tr>
<th></th>
<th>On</th>
<th>Off</th>
<th>Open-loop</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4</td>
<td>+</td>
</tr>
<tr>
<td>Port 3</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Port 4</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>N50Ω</td>
<td>50Ω</td>
<td>50Ω</td>
<td></td>
</tr>
</tbody>
</table>

\[ RR_i = -\frac{V_{GS}}{V_{EXT}} = -\frac{S_{34}}{2} \]

\[ NDF = \prod_{i=1}^{N} (1 + RR_i) \]
Small Signal Analysis: C - Band amplifier

Design specifications:

Band: C

\[ P_{\text{OUT}} > 5 \text{ Watt} \]

AB-class, pulse mode

Pulsed mode

\[ P_{\text{out}} (-3 \text{ dB}) = 38 \text{ dBm} \]

Gain: > 26.5 dB

PAE: 36%

K-Factor: Stable

Oscillation phenomena appear during the characterization step
Small Signal Analysis: C-Band amplifier

Bias: $V_g = -3V$, $V_d = 10V$

Indexing of the transistors

Bias: $V_g = -3V$, $V_d = 3V$

$\text{K-factor + Stability circles:}
\text{conditional Stability}
\text{NDF: Unconditional Stability}$

$9.1\text{GHz} \quad \text{NDF}$

$6.8\text{GHz} \quad \text{NDF(zoom)}$

Two clockwise encircllements...
$F_{osc1} = 6.8$ ???
$F_{osc2} = 9.1$ ???
Small Signal Analysis : C - Band amplifier

Bias : $V_g = -3V$, $V_d = 3V$ (NDi plots)

$NDF = RD_1 \cdot RD_2 \cdot RD_3 \cdot RD_4 \cdot RD_5 \cdot RD_6$

Localization of the unstable mode

$7.2GHz$

$R_{opt}$

$2^{nd}$ stage : a $2^{nd}$ order pole is located near 7 GHz…
Criteria
Stability analysis is performed by using both NDF and K-factor criteria

Implementation
K-factor : Integrated on all Commercially available tools
NDF : should be integrated on Commercially available tools

Advantages
RDₐ can be used to locate an unstable mode \( \rightarrow f_{osc} \) estimated, nature
Stability margins can be directly optimized during the design step \( \rightarrow \) Circuit correction
Statistical studies are possible
It is necessary to have fine transistor modeling
For Black box model, NDF is evaluated at the input using a circulator :

Large Signal Analysis: Perturbed HB equation

Characteristic System: Linearization of the large signal steady state

\[ \Delta V = 0 \]

\( (I - Z(\Omega).U) \Delta V = 0 \)

Nyquist analysis of \( \Delta(j\omega) = \prod_{i=-n}^{n} (\lambda_i) \) ... provided there is no pole / zero cancellation


Reduced characteristic System: defining NDF formulation

\( (I - Z' \times G_M) \times \Delta V_{gs} = 0 \)

Nyquist analysis of \( \text{NDF} = \prod_{i=-n}^{n} (RD_i) \)

\( G_M \) is the conversion matrix of all transistor transconductances

\( RR_i \) is the Return Ratio matrix of the \( i^{th} \) source when \( G_{M_{i-1}} \) \( \ldots \) \( G_{M_i} \) are set to zero

Large Signal Analysis: Open loop analysis

Practical evaluation

Possible states of the transistors

Each frequency position of \( \Omega \) determine a Column of \( R_{Ri} \)

The filter cancels \( k \omega_{in} \pm \Omega \) intermodulation products

H.B (mixer mode) with \( \Omega \in ] DC, \frac{1}{2} F_{in} [ \)
Large Signal Analysis: HBT Ku-band amplifier

**Design Specifications**
- \( F_o \): 12.6 GHz
- Bandwidth: 500 MHz
- \( P_{out} \): 1 W
- C/I: 20dB
- Size: 2.5x4.5 mm²

**Simulation results**
- \( P_{out} \): 30.5 dBm
- PAE: 35%
- K-factor: Stable
- NDF(linear): Stable

**Small-signal analysis**

Nyquist plot of NDF (\( V_{co} = 9V, I_c = 120mA \))

- Bondings ensure a stable state of the bias point

with bondings

without bondings
Large Signal Analysis: HBT Ku-band amplifier

Drain current: 260mA

NDF ($P_{IN} = 19$dBm)

$\frac{f_{in}}{2}$  DC

a half-clockwise encirclement $\rightarrow$ division frequency phenomena

Power spectrum
($P_{IN} = 18.6$dBm)

$\frac{f_{in}}{2}$  $f_{in}$  $3\frac{f_{in}}{2}$

Drain current: 360mA

NDF ($P_{IN} = 20$dBm)

$\frac{f_{in}}{2}$  1.81GHz

a half-clockwise encirclement $\rightarrow$ division frequency
a clockwise encirclement $\rightarrow$ free oscillation

Power spectrum
($P_{IN} = 22.6$dBm)

$\frac{f_{in}}{2}$  0.66GHz

Discret lines Spectrum
Large Signal Analysis : Direct open-loop analysis

Idea : The mathematical utility of the NDF is obtaining a function without poles PRP (undesirable information) representing the zero PRP (required information)

Each element of the RR matrix contains stability information (ie. PRP zeroes)

→ RR matrix can be reduced to its central element RR₀
→ For multistage PAs, a single open-loop analysis is sufficient per stage

However, it is necessary to be able to distinguish the zeros from the poles...

→ Bode’s analysis of ND₀

Nyquist’s criterion is difficult to apply (partial encirclements)

Poles and zeros are distinguished by sweeping a significant parameter (ie. Pᵢⁿ)

Advantages

Low Time consuming (due to high system reduction)
Easy to use
Possible integration to Commercially available tools
Stability margins can be defined

Large Signal Analysis: X-band MMIC amplifier

Design specifications:
- $f_0$: 9.65 GHz
- PAE: max
- Bandwith: $> 150$ MHz
- $P_{out}$: 33 dBm
- $T_{max}$: 60°

Frequency division is observed @ $P_{out} > 29$ dBm

Direct open loop analysis
Large Signal Analysis: \( X\)-band MMIC amplifier

Open-loop analysis: \( \text{ND}_o = f(F_p) \) @ \( F_{\text{in}} \) \( \big|_{P_{\text{in}}} \)

**Graphs:**
- **Magnitude (dB)**
  - Frequency (GHz): 1 to 5
  - Magnitude range: -25 to 15 dB

- **Phasor (°)**
  - Frequency (GHz): 6 to 6
  - Phasor range: -180° to 180°

- **PRP pole (3GHz)**: \( P_{\text{in}} = 10.45 \)
- **PRP zero (\( f_{\text{in}}/2 \))**: \( P_{\text{in}} = 9 \)
Large Signal Analysis: X-band MMIC amplifier

Once the oscillation is found, useless to sweep $f$ ...

\[ ND_0(F_{osc}) = f(P_{in}) \text{ @ } F_{in} = 9.65 \text{ GHz} \]
with $F_{osc} = F_{in}/2$

division phenomena of the first stage

... the entire bifurcation map can be easily computed

Moreover, stability margins can be directly defined...
Oscillator Analysis: Open loop approach

The open loop gain concept

\[ \tilde{G}_{OL}(j\omega) = \text{open loop gain} = \frac{\tilde{V}_1(j\omega)}{\tilde{E}_{ext}(j\omega)} \]
Let’s examine the open loop gain $\tilde{G}_{OL}(j\omega)$ around the frequencies

where

\[ \begin{align*}
&\left|\tilde{G}_{OL}(\omega_o)\right| > 1 \\
&\varphi_{OLG}(\omega_o) = \angle \tilde{G}_{OL}(\omega_o) \approx 0
\end{align*} \]

\[ \Rightarrow \left|\tilde{G}_{OL}\right| > 1 \]

During the transient $\tilde{G}_{OL}$ reduces to a large signal operating point where $\left|\tilde{G}_{LS}(\omega_o)\right| = 1$

\[ \Rightarrow \varphi_{OLG}(\omega_o) = \angle \tilde{G}_{OL}(\omega_o) \approx 0 \]

During the transient $\varphi_{OLG}(\omega)$ shift in frequency to the operating point where:

\[ \varphi_{LS}(\omega_o) = 0 \quad \text{and} \quad \omega_{o\text{large signal}} \approx \omega_{o\text{small signal}} \]
Oscillator Analysis: Simulation results

Linear open-loop gain

MMIC oscillator
(without circuit stabilization)

Gain (dB) Phase (°)

MMIC oscillator
(with circuit stabilization)

Gain (dB) Phase (°)
Linear-loop gain for a transistor oscillator

Are the previous conditions **sufficient** to find a stable oscillation?

**No,** the local stability of the large signal operating point must be analyzed.
Oscillator Analysis: *Bias circuits*

The diagrams show the bias circuits for a high-frequency oscillator. The graphs depict the phase and amplitude characteristics of the oscillator across different frequencies.
A first response can be given by a small signal linear analysis

\[
\begin{align*}
\tilde{G}_{OL}(\omega_o) & > 1 \\
\phi_{OL}(\omega_o) = \angle \tilde{G}_{OL}(\omega_o) & = 0 \\
\left. \frac{d\phi_{OL}}{d\omega} \right|_{\omega_o} & < 0
\end{align*}
\]

Then the final steady-state oscillation should be stable
Oscillator Analysis: Principle

Oscillations start in symmetrical oscillation circuits

In balanced structures two modes of oscillations can be generated:

In phase mode / Out of phase mode

\[ \tilde{G}_{OL}(j\omega) = \text{open loop gain} = \frac{\tilde{V}_1(j\omega)}{\tilde{E}_{1\text{ext}}(j\omega)} \]
Measurement: Predicting parametric oscillations

Hot S parameters

Two-ports model, Linear Time Invariant → Large Signal K-Factor
Measurement: *HBT 6 fingers of 2x30 µm²*

Hot S-parameters extraction @ Z_{ch} n°2

- **Pout versus Pin @ 2.5GHz**
  - `Pout (mW)` vs `Pin (mW)`
  - Graphs for 371.5MHz, 373.5MHz, 375.5MHz

- **Hot S11**
  - Graphs for 371.5MHz, 373.5MHz, 375.5MHz

- **Hot S12**
  - Graphs for 371.5MHz, 373.5MHz, 375.5MHz

- **Hot S21**
  - Graphs for 371.5MHz, 373.5MHz, 375.5MHz

- **Hot S22**
  - Graphs for 371.5MHz, 373.5MHz, 375.5MHz
Measurement: *TBH 6 fingers of 2x30 µm²*

- **Measurement:**
  - TBH 6 fingers of 2x30 µm²
  - Spectrum @ f
  - Phase (ΓLOAD * ΓS)

- **Graphs:**
  - P_{out} @ F=2.5GHz
  - Spectrum @ f
  - P_{in} (mW)

- **Equations and Notes:**
  - \[ P_{out} = 79\, \text{mW} \]
  - Frequency f (MHz)

- **References:**
Measurement: $LSNA + HIP$

Applications for MMICs validation & Stability

Class-F PA

$P_{in} = 1.8\,\text{GHz} - 2\text{HBT AsGA}$

HIP enables easy detection of oscillations

LSNA enables phase measurements

Knowledge of nonlinear phenomena & Optimal design of PA

Measurement: Parametric stability

PERIODIC
- Fundamental Harmonics
- Rational Frequency division
  \[ r = \frac{\omega_1}{\omega_2} \in \mathbb{Q} \]

QUASI-PERIODIC
- Non rational Frequency division
  \[ r = \frac{\omega_1}{\omega_2} \notin \mathbb{Q} \]

CHAOTIC
- Non periodic

Phase space
- Limit Cycle
- Torus
- Strange attractor

Spectra
- Fundamental Harmonics
- Rational frequency division
- Non rational frequency division

Non-linear Ordinary Differential Equations
\[ F_{NL}(x, \dot{x}, y, ...) = 0 \]

Microwave NonLinear Circuit
Non-linear Dynamic System

\[ \omega_1, \omega_2 \]

\[ r \in \mathbb{Q} \]

\[ r \notin \mathbb{Q} \]
Measurement: \(LSNA + HIP\)

Experimental verification of the oscillation mode
Large device count $\rightarrow$ thousands of transistors...

Parasitic extraction $\rightarrow$ tens of thousands of nodes...

User implemented NDF and probing technique become impractical
- Cumbersome (large number of devices)
- Insecure (extensive number of internal loops)

Stability analysis based on Full nodal equation is necessary
- Numerical techniques for full nodal stability have matured
- May not be implemented by the designer
- Now available in some commercial CAD tools
**Modified nodal perturbation equation** \( Y(p)V(p) = 0, \quad p = \sigma + j\omega \)

**Two approaches** → Nyquist method or Eigen value method

**Nyquist method** → Generalized NDF

\[
D(p) = \frac{\det[Y(p)]}{\det[Y_p(p)]}
\]

\[
V(p) = \begin{bmatrix} V_A(p) \\ V_p(p) \end{bmatrix} \rightarrow \text{transistors nodes (active)} \quad \rightarrow \quad Y(p) = [Y_A(p) \mid Y_p(p)]
\]

\[
\rightarrow \text{remaining nodes (passive)}
\]

**Investigate the complex phase trajectory** → \( D(j\omega) / |D(j\omega)| \)

The number of clockwise encirclements indicate instability and associated frequencies

**Limitation** → Pole-zero cancelation in determinant evaluation may hide some instability

**CAD tools** → Ansoft / Nexxim, Xpedion / GoldenGate
**Full nodal stability analysis:** Nyquist method

**Nyquist method** \(\rightarrow\) generalized NDF

\[
D(p) = \frac{\det[Y(p)]}{\det[Y_p(p)]} = |D(p)|e^{j\phi_{D(p)}}
\]

Investigate the complex phase trajectory \(\rightarrow\) \(D(j\omega)/|D(j\omega)|\)

*The number of clockwise encirclements indicate instability and associated frequencies*

**Limitation** \(\rightarrow\) Pole-zero cancelation in determinant evaluation may hide some instability

**CAD tools** \(\rightarrow\) Ansoft/Nexxim, Xpedion/GoldenGate
Full nodal stability analysis: Eigen value method

Eigen value method
\[ Y(p)V(p) = 0, \quad p = \sigma + j\omega \]

Most RF circuits are modeled by R-L-C networks
\[ Y(p) = [G + pC] \]

Unstable circuit if Eigen values \( \lambda \) on the half right plane

Imaginary part \( j\omega \) of eigen value gives the frequency of instability

\[ \frac{d\sigma}{d\eta} \text{ of eigen value gives a notion of stability margin versus the parameter } \eta \]

Eigen value problem
\[ AV = -\lambda V, \quad A = G^{-1}C, \quad \lambda = p^{-1} = \frac{\sigma - j\omega}{\sigma^2 + \omega^2} \]

Efficient Krylov based numerical techniques and tool boxes available can solve very large systems

Limitation → Numerical instability for large problem size; apply well only to DC operating point stability

CAD tools → Xpedion/GoldenGate, Mentor/EldoRF
Full nodal stability analysis: \( PCS \text{ chain} \)

1200 MOS + BJT

8000 nodes – extracted view

Nyquist method: 8 mn

Eigen value method: 4 mn

Unstable eigen value: \([2 + j0.3]\) GHz

Parasitics creates a low frequency instability

Instability freq: 0.2 GHz

Transient simulation shows the 265 MHz low frequency oscillation
Simulation guide for designer

Small signal $\rightarrow$ S parameters $\rightarrow$ K-factor + NDF

Large signal @ $F_{in}$ $\rightarrow$ HB (mixer mode) $\rightarrow$ $\text{ND}_o = f (F_p) + Bode's analysis$

A simple and efficient way to detect and optimize stability as a common parameters in a CAD commercialy tool (Stability margins are defined)

Open loop analysis is also the best way to design autonomous circuits (oscillator)

It’s always necessary to have fine transistor modeling...

Measurement sets-up...

Hot S parameters $\rightarrow$ for predicting parametric oscillation (Large signal K-factor)

LSNA + HIP $\rightarrow$ offers an unique experimental verification of even and odd oscillation modes

...essential for the analysis of both operating conditions and reliability aspects

RF SOC problematic

Full Nodal Analysis $\rightarrow$ Nyquist / Eigenvalue methods should be integrated in CAD utilities